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Internal waves around a body moving in a compressible density-stratified fluid

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A body is started from rest and moves on an arbitrary path in an inviscid isothermal compressible atmosphere. The phase configuration of the internal waves and the gravity-modified acoustic waves which are generated by the body is studied using small amplitude wave theory. When the body moves at supersonic speeds and the background density gradient approaches zero, it is shown how the wave solutions approach the pure acoustic wave solutions of Lilley *et al.* (1953).

1. Introduction

Recent experimental evidence indicates that acoustic gravity waves can be generated in the atmosphere by various natural and artificial sources such as earthquakes, severe weather fronts and nuclear explosions (see Liu & Yeh 1972). Oscillatory waves under the combined action of gravity and compressibility have been discussed in detail by Moore & Spiegel (1964) and by Lighthill (1967). Liu & Yeh (1972) considered the theory for point disturbances travelling in a horizontal plane with constant velocity in an isothermal atmosphere. Solutions for the waves around a two-dimensional body moving with a constant velocity at an angle to the horizontal have been evaluated by Rarity (1973) and solutions involving a superimposed forced oscillation by Silcock (1973).

In the present paper, equations are given for the phase configuration of the waves about a point disturbance moving on an arbitrary path in an isothermal atmosphere. A superimposed oscillation of the forcing region is included in the analysis. The theory for the Cauchy–Poisson waves due to an impulsive start is also given. The radiation condition, which states that energy must propagate away from the disturbance, is included in the analysis so that a study of the wave-number surfaces is not required explicitly. The general equations are studied in greater detail for several specific cases. One such case considers the wave pattern around a two-dimensional body moving on a circular path at supersonic speed, and it is shown how the waves approach the pure acoustic waves which would occur in a homogeneous fluid (Lilley *et al.* 1953). Another, much simpler case is the wave pattern around a point disturbance moving at constant velocity in an incompressible fluid. These waves are shown to compare well with schlieren photographs of a sphere moving in stratified brine.

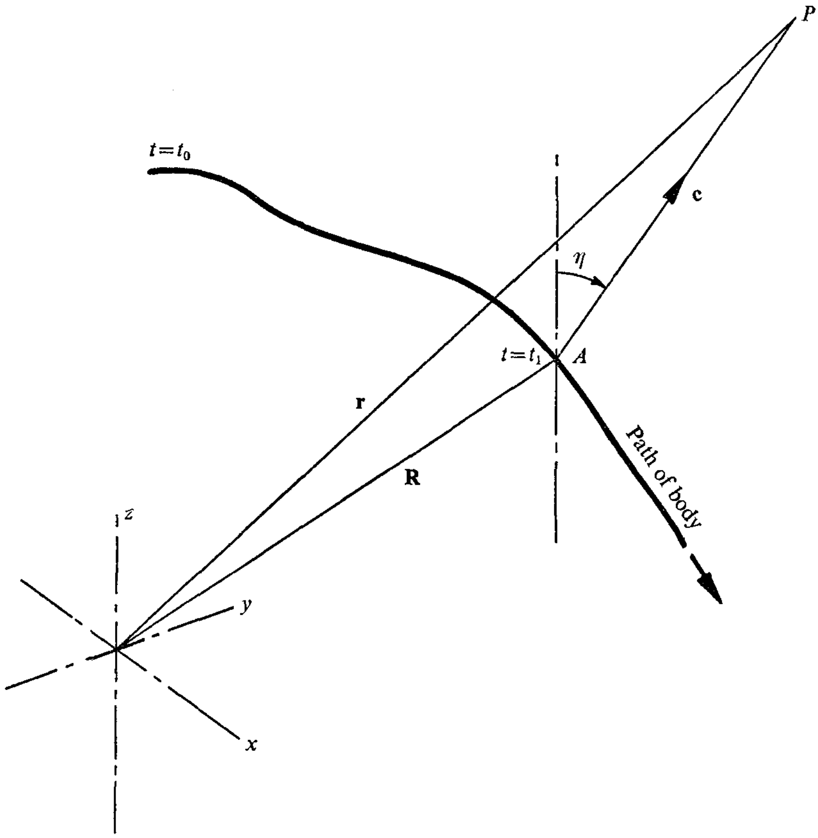


FIGURE 1. The path of the body.

2. Analysis

2.1. *Waves around a moving body*

We consider an isothermal atmosphere which has a constant buoyancy frequency N given by

$$N = \left(-\frac{g}{\rho_0}\frac{d\rho_0}{dz} - \frac{g^2}{a_0^2}\right)^{\frac{1}{2}} = (\gamma-1)^{\frac{1}{2}}\frac{g}{a_0}, \tag{1}$$

where $\rho_0(z)$ is the density of the undisturbed fluid, z is the vertical co-ordinate, g is the gravitational acceleration, a_0 is the sound speed and γ is the ratio of the specific heats. The dispersion relation for a point disturbance (Hines 1960) is

$$\omega^4 - \omega^2 a_0^2\left[k_1^2 + k_2^2 + k_3^2 + \frac{\gamma^2}{4(\gamma-1)}\frac{N^2}{a_0^2}\right] + N^2 a_0^2(k_1^2+k_2^2) = 0, \tag{2}$$

where ω is the frequency associated with the energy propagating from the disturbance and (k_1, k_2, k_3) are the components of the wavenumber $\mathbf{k}$ in the (x, y, z) directions (see figure 1). The x, y plane is horizontal. [If $a_0 \rightarrow \infty$ then this equation reduces to the incompressible dispersion relation

$$\omega^2[k_1^2+k_2^2+k_3^2] = N^2(k_1^2+k_2^2),$$

where N^2 is now $-g\rho_0^{-1} d\rho_0/dz$. Throughout the following analysis the incompressible results can be obtained by allowing $a_0 \to \infty$.]

At sufficiently large distances from the forcing region, the group velocity $\mathbf{c}$ and the phase velocity $\mathbf{v}_p$ are given by (see, for instance, Lighthill 1965) $\mathbf{c} = \nabla_k \omega$ and

$$\mathbf{v}_p = (u, v, w) = \omega(k_1^2 + k_2^2 + k_3^2)^{-1} \{k_1, k_2, k_3\}.$$

The direction of the phase velocity is defined by the angles ψ and η such that

$$(u, v, w) = |(u^2+v^2)^{\frac{1}{2}}| \{\cos\psi, \sin\psi, -\tan\eta\}. \tag{3}$$

The group velocity, evaluated from the dispersion relation, is given by

$$\frac{\mathbf{c}}{a_0} = \frac{\alpha M}{M^2G^2-\alpha^2} \{(G^2-1)\cos\psi, (G^2-1)\sin\psi, -G^2\tan\eta\}, \tag{4}$$

where $$G = \omega/N, \quad \alpha = \frac{a_0 M}{G^2N^2}(k_1^2+k_2^2+k_3^2)\,|(u^2+v^2)^{\frac{1}{2}}|.$$

M is the Mach number Q/a_0, where Q is the speed of the body. It is assumed that the group-velocity concept can be applied to the whole flow field. Schlieren photographs have shown this to be a reasonable assumption in stratified brine (Stevenson 1973). The analysis is simplified slightly if formulated in terms of the phase-velocity direction (ψ, η) rather than in terms of the wavenumber direction, used in previous papers. This is because energy propagating in a particular direction has only one phase-velocity direction but it could have two wavenumber directions which differ by π. The dispersion relation can now be written in the dimensionless form

$$M^2(G^2-C) - \alpha^2(G^2\sec^2\eta - 1) = 0, \tag{5}$$

where $$C = \gamma^2[4(\gamma-1)]^{-1}.$$

The theory now follows that of Stevenson (1973). A body starts to move at $t = t_0$ with velocity $\mathbf{Q}(t)$ such that $\mathbf{R}(t)$ is its distance from an origin O which is fixed in the undisturbed background fluid, as in figure 1. The body is at point A when $t = t_1$, and energy radiated from this point reaches point P, which is at a distance $\mathbf{r}(t)$ from the origin, at time t. Thus

$$\mathbf{r}(t) = \mathbf{R}_1 + (t-t_1)\,\mathbf{c}, \tag{6}$$

where the subscript one refers to conditions at time t_1. The phase at P is given by

$$\Phi(t) = (\mathbf{k}.\mathbf{c} - \omega)(t-t_1) - \omega_f t_1 + \phi_0, \tag{7}$$

where ϕ_0 is a constant and ω_f is a forcing frequency associated with the body. The relation between ω_f and ω is given by the Doppler equation

$$\omega = \omega_f + \mathbf{Q}_1.\mathbf{k}. \tag{8}$$

Any number of forcing frequencies may be included simply by superimposing the waves due to each individual value of ω_f. Equation (7) is rearranged to give

$$N(t-t_1) = (\mathbf{k}.\mathbf{c}/N - G + B)^{-1}\phi(t), \tag{9}$$

where $B = \omega_f/N$ and $\phi(t) = \Phi(t) - \phi_0 + \omega_f t$. If $\mathbf{k}$ is eliminated from (9) then

$$N(t-t_1) = \frac{(M^2G^2-\alpha^2)\,\phi(t)}{G(\alpha^2-M^2C)+B(M^2G^2-\alpha^2)}. \tag{10}$$

The radiation condition implies that $t - t_1 > 0$.

The phase configuration of the waves generated by a body moving on an arbitrary path may be determined from the above equations. Considerable simplification occurs, however, if the body moves in the x, z plane such that $\mathbf{Q} = (Q\cos\theta,\ 0,\ Q\sin\theta)$, where θ is the angle which the path of the body makes with the horizontal. In this case the Doppler equation (8) becomes

$$G-B = \alpha G(\cos\theta_1\cos\psi - \sin\theta_1\tan\eta), \tag{11}$$

where $\theta_1 = \theta(t_1)$. The expressions for $t-t_1$, equation (10), and for $\mathbf{c}$, equation (4), are substituted into (6) to obtain an equation for the phase configuration. When θ_1 is not zero, η can be eliminated from this equation using (11), so that

$$N\mathbf{r}/Q = N\mathbf{R}_1/Q - \{G(\alpha^2-M^2C)+B(M^2G^2-\alpha^2)\}^{-1}\{\alpha(1-G^2)\cos\psi,\ \alpha(1-G^2)\sin\psi,\ G(B-G)\operatorname{cosec}\theta_1+\alpha G^2\cot\theta_1\cos\psi\}\,\phi(t), \tag{12}$$

where, from (5) and (11),

$$\alpha = \{(1-G^2)\sin^2\theta_1 - G^2\cos^2\theta_1\cos^2\psi\}^{-1}[G(B-G)\cos\theta_1\cos\psi \pm \sin\theta_1\{(G-B)^2(1-G^2)+M^2(C-G^2)((1-G^2)\sin^2\theta_1-G^2\cos^2\theta_1\cos^2\psi)\}^{\frac{1}{2}}]. \tag{13}$$

When $\theta_1 = 0$, η can be eliminated from the equation for $\mathbf{r}$ using the dispersion relation (5), so that the equation for the phase configuration becomes

$$\frac{N\mathbf{r}}{Q} = \frac{N\mathbf{R}_1}{Q} - \{G(\alpha^2-M^2C)+B(M^2G^2-\alpha^2)\}^{-1}\{\alpha(1-G^2)\cos\psi,\ \alpha(1-G^2)\sin\psi,\ \pm G[G^2(M^2-\alpha^2)-(M^2C-\alpha^2)]^{\frac{1}{2}}\}\,\phi(t), \tag{14}$$

where, from (11), $$\alpha = (G-B)/G\cos\psi. \tag{15}$$

$\phi(t)$ varies by 2π between one wave crest and the next. The incompressible results can be obtained by writing $M = 0$ in (12)–(15).

The radiation condition implies that $t_0 \leqslant t_1 \leqslant t$, and if the body stops at time $t_2 < t$, then $t_1 \leqslant t_2$.

2.2. *Impulsive waves*

When the wave system around an impulsive disturbance is considered, then the wavenumbers are only restricted to those which satisfy the dispersion relation. If the disturbance occurs at $t = 0$ then, from (6), $\mathbf{r} = \mathbf{c}t$. η is eliminated from the expression for $\mathbf{c}$, equation (4), using the dispersion relation, so that

$$\frac{N\mathbf{r}}{a_0} = \frac{\alpha MNt}{M^2G^2-\alpha^2}\left\{(G^2-1)\cos\psi,\ (G^2-1)\sin\psi,\ \pm\frac{G}{\alpha}[G^2(M^2-\alpha^2)-(M^2C-\alpha^2)]^{\frac{1}{2}}\right\}. \tag{16}$$

Equation (10), after rearranging, becomes

$$\alpha^2 = M^2\{G^2 - G(G^2-C)(G+\phi/Nt)^{-1}\}. \tag{17}$$

The equations for the phase configuration will be considered in some detail for several cases in the next section.

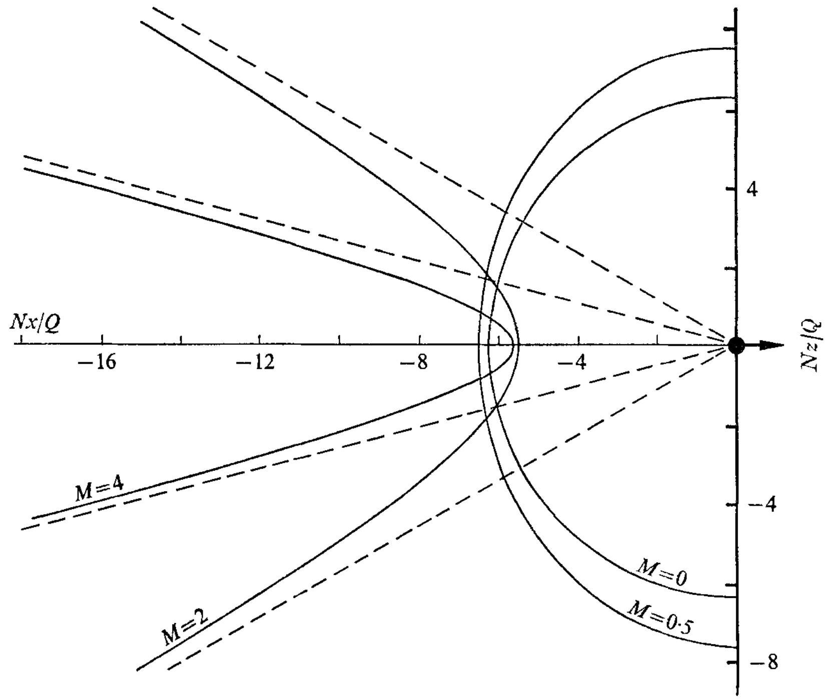


FIGURE 2. Two-dimensional waves when the body moves horizontally with various Mach numbers. $\phi = 2\pi$. – – –, Mach wedges.

3. Specific solutions

3.1. *A body moving horizontally*

The first case that will be considered is the two-dimensional wave pattern about a horizontal cylinder moving horizontally with constant velocity. If the body has been travelling for an infinite time we let $t_0 = -\infty$ and put $\mathbf{R}_1 = (Qt_1, 0)$ and $\psi = 0$ in (14). In order to evaluate the phase configuration relative to the body we write $(X, Z) = \mathbf{r} - (Qt, 0)$ so that, from (10), (14) and (15),

$$N(X, Z)/Q = -\{G(\alpha^2 - M^2C) + B(M^2G^2 - \alpha^2)\}^{-1}\{\alpha(1 - G^2) + (M^2G^2 - \alpha^2), \pm G[G^2(M^2 - \alpha^2) - (M^2C^2 - \alpha^2)]^{\frac{1}{2}}\}\,\phi(t), \quad (18)$$

where $$\alpha = (G - B)/G. \quad (19)$$

These equations have been used to calculate the steady waves, for several Mach numbers, shown in figure 2. A value of 2π has been used for ϕ and, in this and subsequent examples, $\gamma = 1{\cdot}4$ and $C = 1{\cdot}225$. When $M = 0$ a line of constant phase is a semicircle behind the cylinder: the usual lee-wave solution. When M is between zero and $C^{-\frac{1}{2}}$, a line of constant phase is a semiellipse with the major axis vertical. The wave spacing, the distance between one wave crest and the next with the same wavenumber, increases as M increases and becomes infinite when $M = C^{-\frac{1}{2}}$. There are no waves if $C^{-\frac{1}{2}} < M \leqslant 1$. For $M > 1$ the waves are hyperbolas with the Mach wedge as asymptote.

Oscillatory waves for various forcing frequencies at a Mach number of 2 are

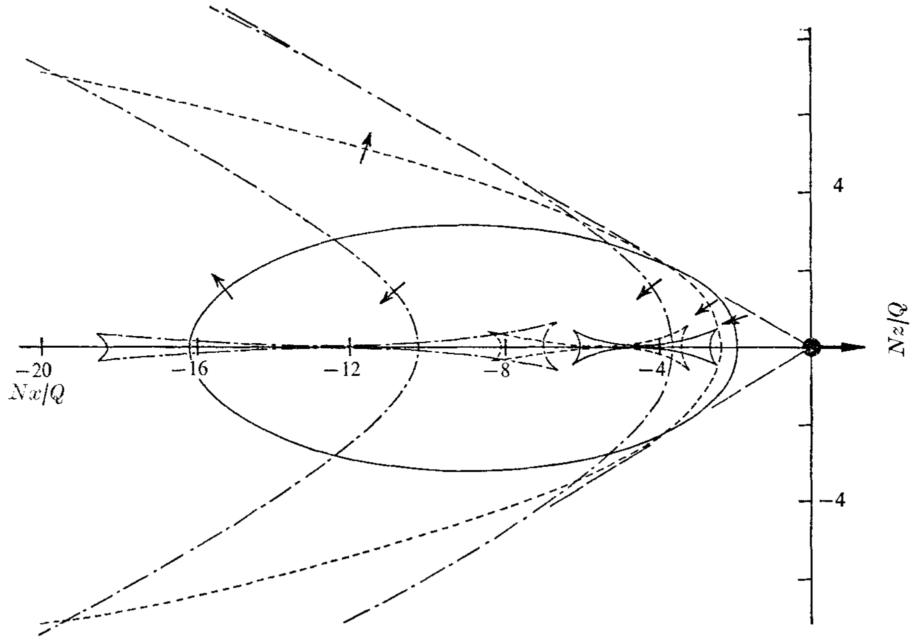


FIGURE 3. Two-dimensional oscillatory waves when the body is moving horizontally at $M = 2$. The arrows indicate the direction in which the lines of constant phase move relative to the body. $\phi = 2\pi$. —·—; $B = 0{\cdot}5$; --,-, $B = C^{\frac{1}{2}}$; ——, $B = 1{\cdot}5$; — —, Mach wedge.

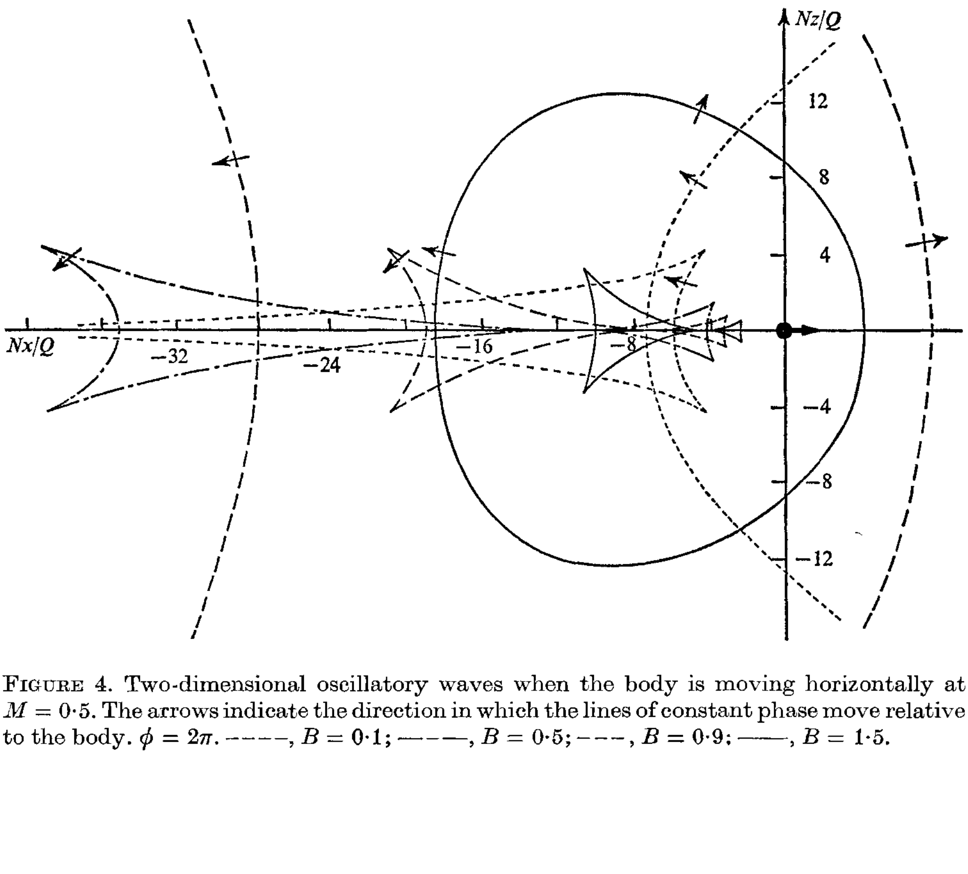


FIGURE 4. Two-dimensional oscillatory waves when the body is moving horizontally at $M = 0{\cdot}5$. The arrows indicate the direction in which the lines of constant phase move relative to the body. $\phi = 2\pi$. ----, $B = 0{\cdot}1$; — — —, $B = 0{\cdot}5$; - - -, $B = 0{\cdot}9$; ——, $B = 1{\cdot}5$.

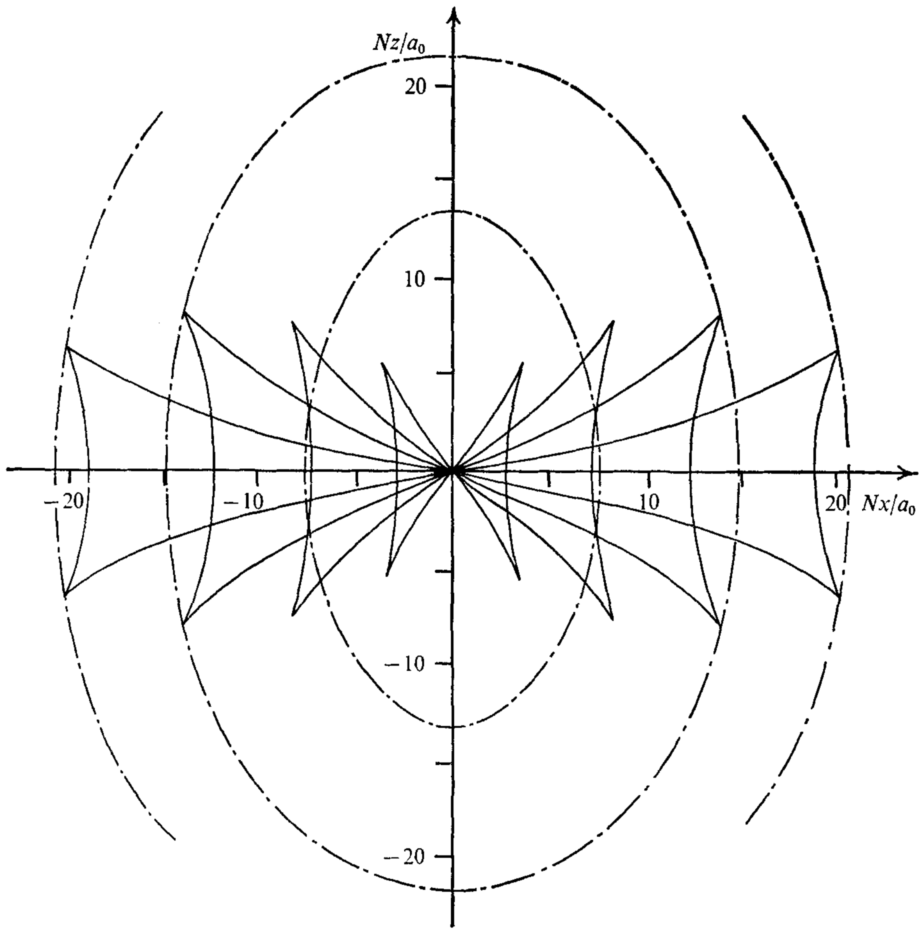


FIGURE 5. Impulsive-start waves. ———, acoustic waves. $Nt = 10\pi$; $\phi = 2n\pi$ with $n = 1, 2, 3, 4$.

shown in figure 3. Closed internal waves with four cusps occur at all frequencies, and a pair of acoustic waves is present when $0 < |B| < C^{\frac{1}{2}}$. As $|B| \rightarrow C^{\frac{1}{2}}$ the acoustic waves approach one another, and they merge and become a closed curve passing through $(-\infty, 0)$ when $|B| = C^{\frac{1}{2}}$. The closed acoustic wave decreases in size as $|B|$ increases and is shown in figure 3 for $B = 1{\cdot}5$. All the waves are within the Mach wedge.

Figure 4 shows oscillatory waves for several values of B at $M = 0{\cdot}5$. For small $|B|$ the waves are like the incompressible waves described by Stevenson & Thomas (1969). At larger $|B|$ acoustic waves are present and behave like those for $M = 2$ except of course that they can propagate ahead of the body and are not bounded by a Mach wedge.

3.2. *Cauchy–Poisson waves*

The equations of § 2.2 with $\psi = 0$ have been used to evaluate the two-dimensional impulsive waves shown in figure 5. The cusped internal waves are formed near the z axis and move towards higher $|x|$. These impulsive waves were discussed

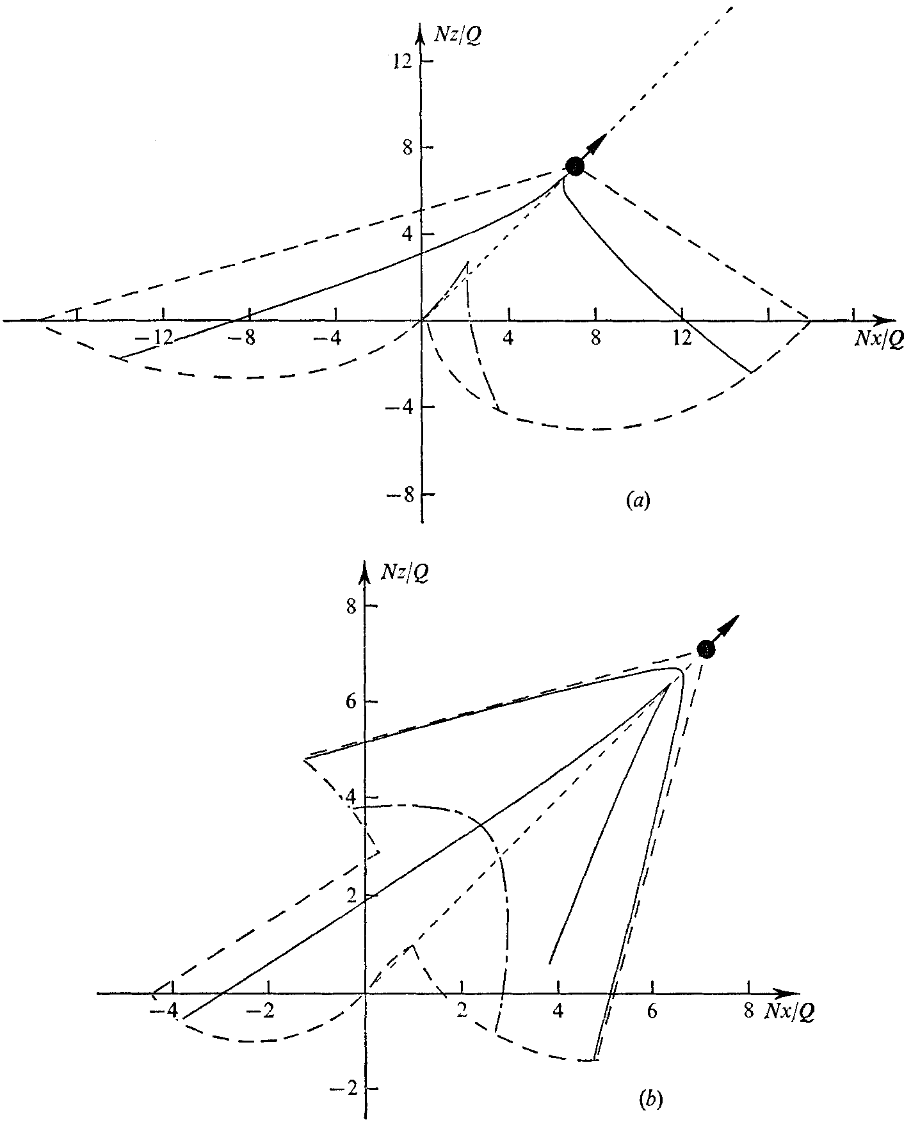


FIGURE 6. A body which started from the origin moves with constant velocity at 45° to the horizontal. $Nt = 10$. ——, waves with $\phi = \frac{1}{4}\pi$; — — —, waves with $\phi = \frac{9}{4}\pi$; - - -, locus of energy. (a) $M = 0{\cdot}5$. (b) $M = 2{\cdot}0$.

by Cole & Greiffinger (1969) in a study of atmospheric waves produced by an earthquake or nuclear explosion. In the next subsection it will be shown how the impulsive waves merge with the steady wave system.

3.3. *The steady wave system which develops after a body starts from rest*

Rarity (1973) and Silcock (1973) have discussed the wave system around a two-dimensional body which has been travelling at an angle to the horizontal with constant velocity for an infinite time. Consequently only the waves around a

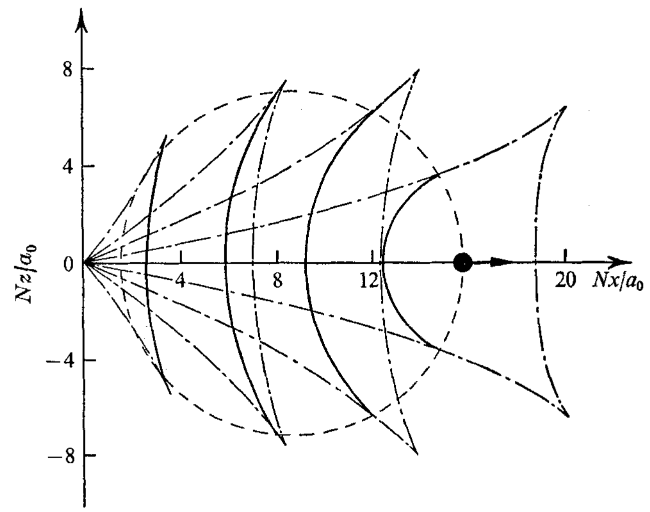


FIGURE 7. Steady horizontal motion of a body which started impulsively at the origin. $Nt = 10\pi$; $M = 0{\cdot}5$; $\phi = 2n\pi$ with $n = 1, 2, 3, 4$. ——, 'steady' internal wave system; – – – –, locus of energy of steady wave system; — — —, impulsive waves. Only the impulsive wave system for $x > 0$ is shown.

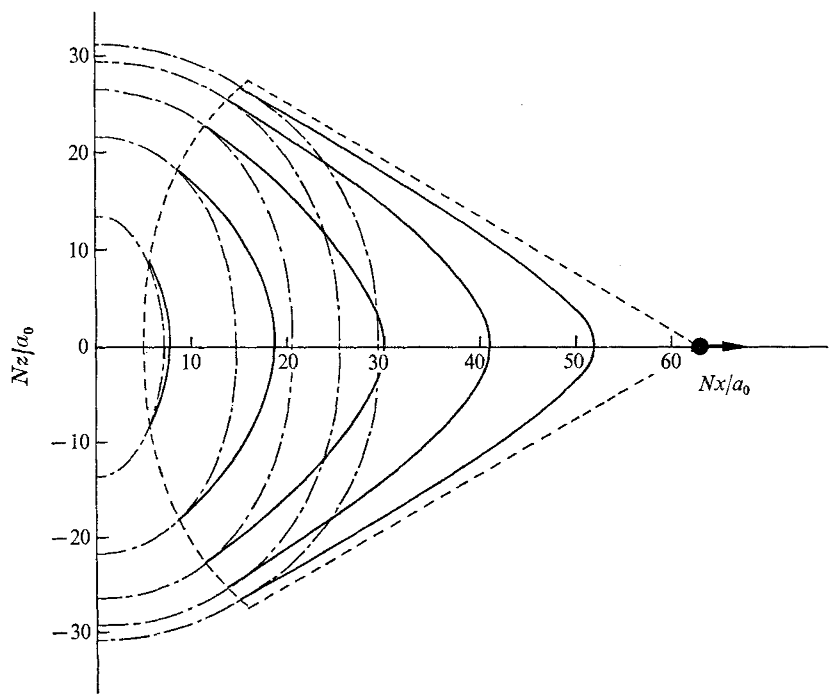
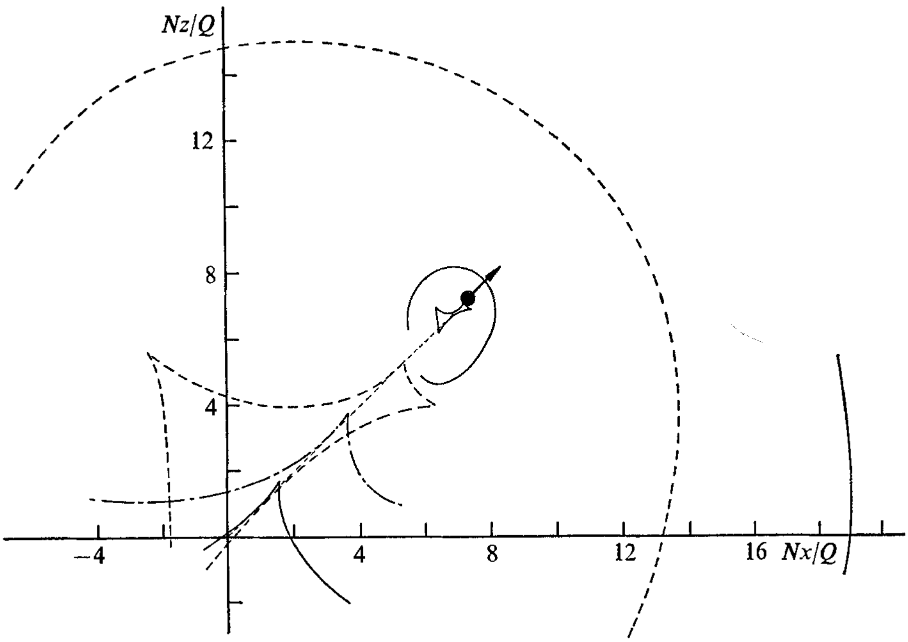
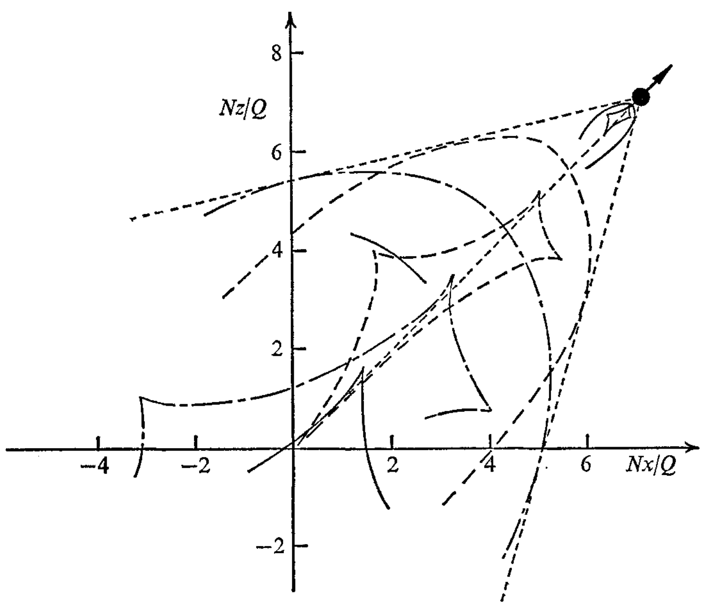


FIGURE 8. Steady horizontal motion of a body which started impulsively at the origin. $M = 2{\cdot}0$; $\phi = 2n\pi$ with $n = 1, 2, \ldots, 5$. ——, steady acoustic wave system; – – – –, locus of energy of steady wave system; — — —, acoustic impulsive waves. Only the impulsive wave system for $x > 0$ is shown.



(a)



(b)

FIGURE 9. Two-dimensional oscillatory waves when the body is moving at 45° to the horizontal, having started from the origin. $B = 1{\cdot}5$; $\omega_c t = 10$; $\phi = \frac{1}{4}\pi + 2n\pi$ with $n = 0, 1, 2, 3$. (a) $M = 0{\cdot}5$. (b) $M = 2{\cdot}0$, showing the Mach wedge (- - - - - -).

body which has been moving for a finite time will be presented here. Equations (12) and (13) with $\psi = 0$ have been used to evaluate the waves shown in figure 6. The body started to move at the position $x = z = 0$ and travelled at 45° to the horizontal at Mach numbers of 0·5 and 2·0. When $M > 1$ the acoustic waves again have the Mach wedge as asymptote. The energy envelopes, the boundaries within which energy is to be found, are also shown in the figure.

The way in which the waves merge with the impulsive-start waves is shown in figures 7 and 8 for Mach numbers of 0·5 and 2·0 with $\theta = 0°$. The impulsive waves are tangential to the initial part of the moving-body waves.

Examples of the oscillatory wave system with a frequency ratio of 1·5 are shown in figures 9 (*a*) and (*b*), where $M = 0{\cdot}5$ and 2·0 respectively and $\theta = 45°$.

3.4. *Three-dimensional waves around a sphere*

Equations (12)–(15) with $M = 0$ have been used to evaluate the incompressible waves shown in figures 10 and 11 (plates 1 and 2). The schlieren photographs in these figures show the waves which develop around a moving sphere. The photographs were taken using the experimental equipment described by Stevenson (1973). There is reasonable agreement between the theory and the experiments, except in the case of figures 10 (*e*) and (*f*), which show a large sphere, with a complicated wake, producing its own oscillatory wave system.

Figures 12 and 13 illustrate several compressible solutions for the waves around a point disturbance moving horizontally. As the solutions are symmetric about the line of motion, only half of the wave system is shown. Solutions in planes y = constant are given above the x axis and those in planes z = constant below the x axis.

3.5. *A circular path in a vertical plane*

A two-dimensional body starts from a position $(-R, 0)$ and travels on a circular path of radius R with angular frequency ω_c, such that $\omega_c R = Q$. Thus

$$\mathbf{R}_1 = (-\cos\omega_c t_1, \sin\omega_c t_1)R \quad \text{and} \quad Q = (\sin\omega_c t_1, \cos\omega_c t_1)\,Q.$$

With $\psi = 0$ equations (12) and (13) become

$$\left(\frac{x}{R}, \frac{z}{R}\right) = (-\cos\omega_c t_1, \sin\omega_c t_1) - \frac{\omega_c}{N}\{G(\alpha^2 - M^2C) + B(M^2G^2 - \alpha^2\}^{-1} \times \{\alpha(1-G^2), G(B-G)\sec\omega_c t_1 + G^2\alpha\tan\omega_c t_1\}\,\phi, \quad (20)$$

where

$$\alpha = \frac{G(B-G)\sin\omega_c t_1 \pm \cos\omega_c t_1\{(G-B)^2(1-G^2) + M^2(C-G^2)(\cos^2\omega_c t_1 - G^2)\}^{\frac{1}{2}}}{\cos^2\omega_c t_1 - G^2}. \quad (21)$$

These are solved in conjunction with (10) and the radiation condition. Figures 14 and 15 show the waves that develop when a body travels at Mach numbers of 0·5 and 2·0 with an angular frequency ω_c of $0{\cdot}1N$. Figure 15 also shows the weak shock waves which would occur if there was no stratification (Lilley *et al.* 1953). Figure 16 shows an oscillatory system with $M = 0{\cdot}5$ and $B = 0{\cdot}5$.

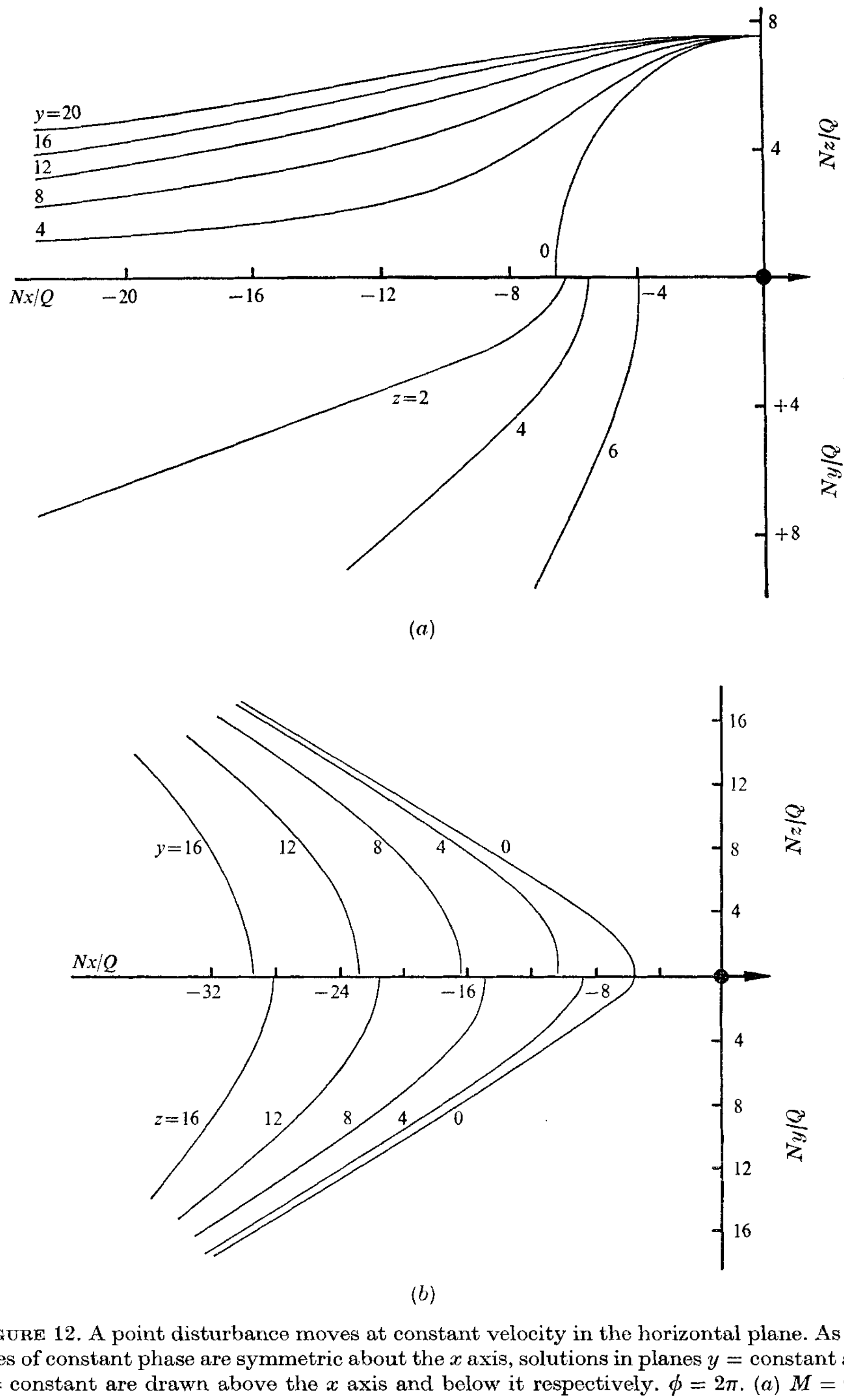


FIGURE 12. A point disturbance moves at constant velocity in the horizontal plane. As the lines of constant phase are symmetric about the x axis, solutions in planes y = constant and z = constant are drawn above the x axis and below it respectively. $\phi = 2\pi$. (a) $M = 0{\cdot}5$. (b) $M = 2{\cdot}0$.

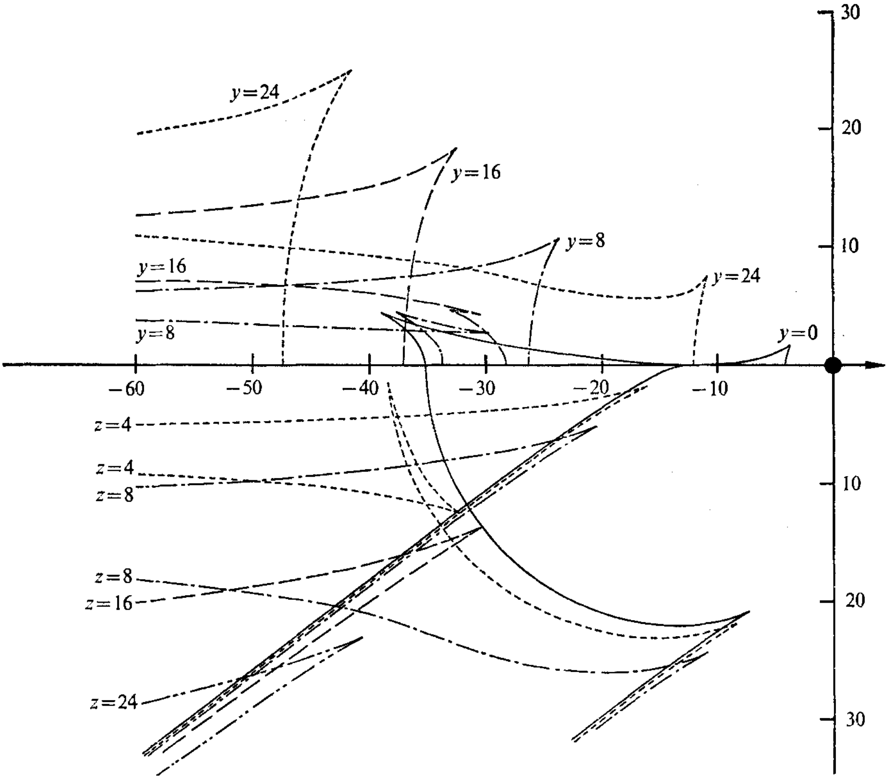


FIGURE 13. An oscillating point disturbance moves with a constant mean velocity in the horizontal plane. Solutions in planes y = constant and z = constant are drawn above and below the x axis respectively. $M = 0{\cdot}5$; $\phi = 2\pi$; $B = 0{\cdot}5$.

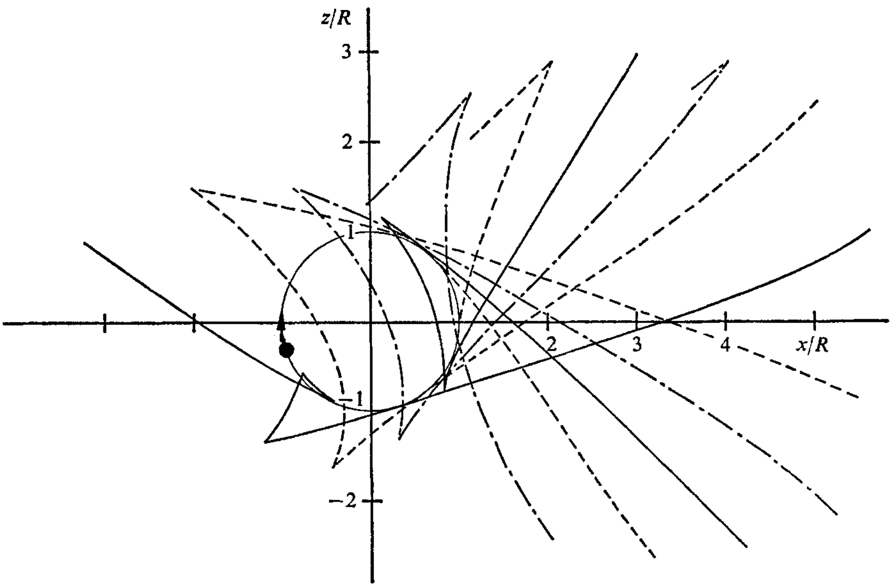


FIGURE 14. A two-dimensional body moves on a circular path in the x, z plane with a constant angular velocity, having started from $(-R, 0)$. $\omega_c t = 6{\cdot}0$; $\omega_c/N = 0{\cdot}1$; $\phi = (2n+1)\pi$ with $n = 0, 1, \ldots, 5; M = 0{\cdot}5$.

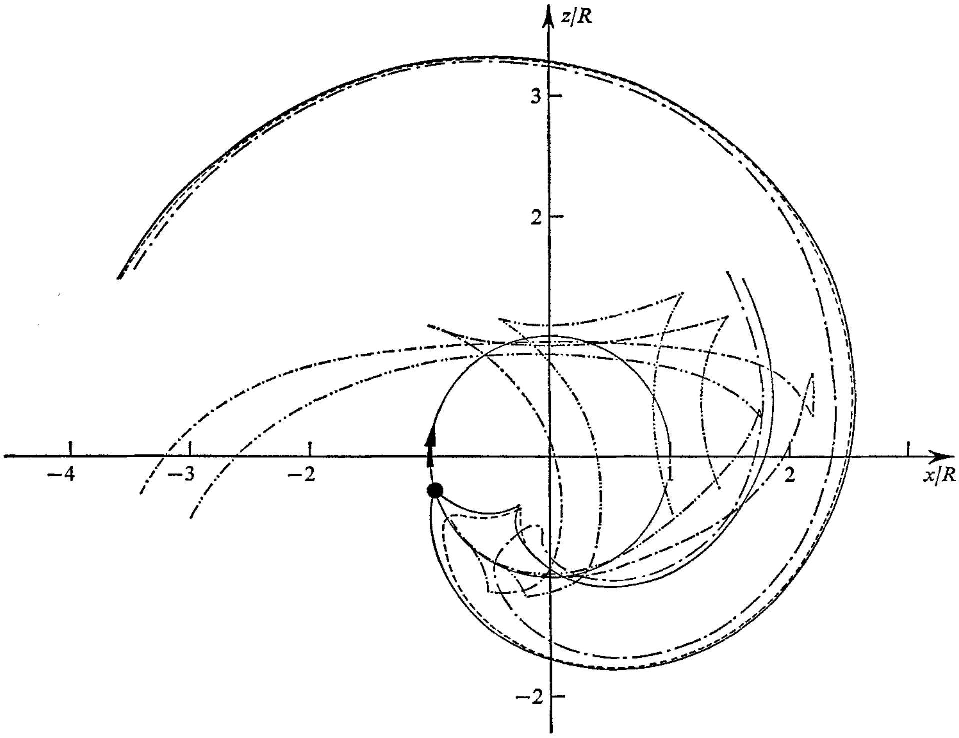


FIGURE 15. A two-dimensional body moves on a circular path in the x, z plane with a constant angular velocity, having started from $(-R, 0)$, $\omega_c t = 6\cdot0$; $\omega_c/N = 0\cdot1$; $M = 2\cdot0$, —·—·, – – – –, (acoustic) $\phi = \pi$; —··—··—, —·—· (acoustic) $\phi = 3\pi$; ——, weak shock position.

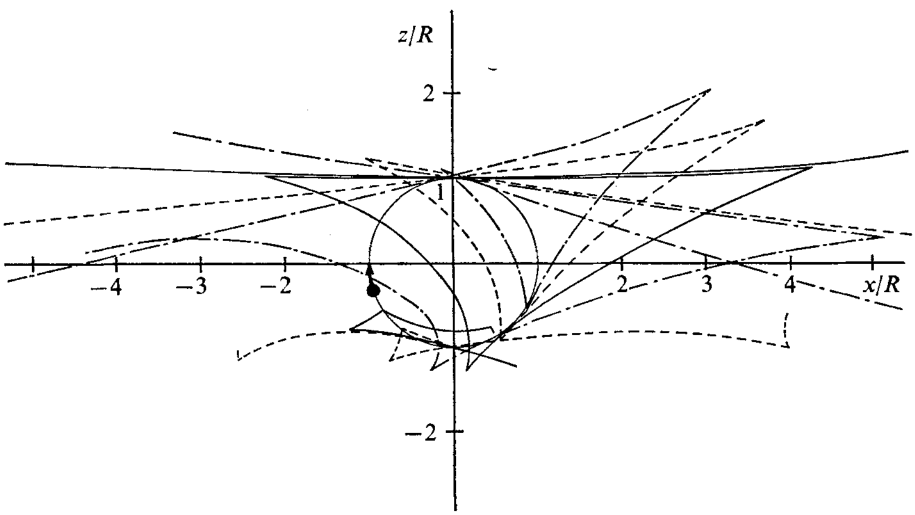


FIGURE 16. A two-dimensional oscillating body moves on a circular path in the x, z plane with a constant mean angular velocity, having started from $(-R, 0)$. $\omega_c t = 6\cdot0$; $\omega_c/N = 0\cdot1$; $M = 0\cdot5$; $B = 0\cdot5$; $\phi = (2n+1)\pi$ with $n = 0, 1, \ldots, 5$.

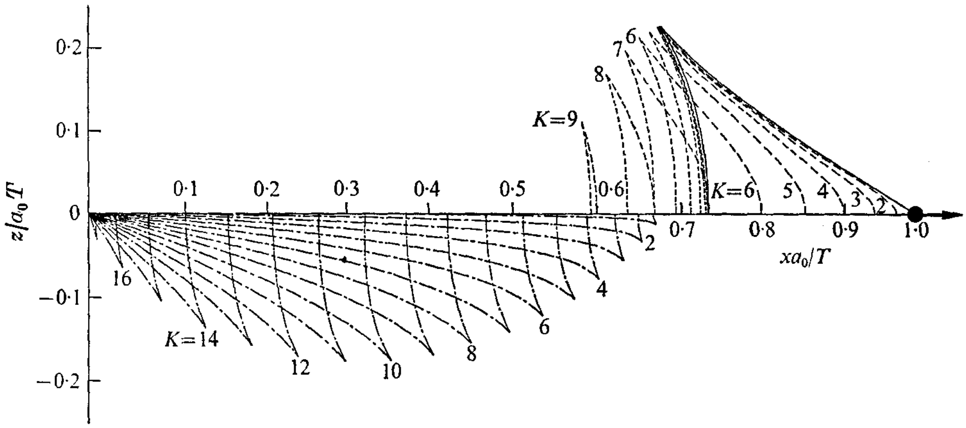


FIGURE 17. A two-dimensional body, starting from rest, moves with constant acceleration in a horizontal plane until $M = 2\cdot0$. As the lines of constant phase are symmetric about the x axis, only half of each wave system is shown (acoustic waves above the axis). ——, weak-shock position.

The positions of the weak shock waves are obtained from the equations in this paper by letting $\phi/N^2 \to 1$ as $N \to 0$ with $B = 0$. In the case of the circular path

$$\left(\frac{x}{R}, \frac{z}{R}\right) = (-\cos\omega_c t_1, \sin\omega_c t_1) + \frac{\alpha}{M^2}(\omega_c t - \omega_c t_1)\left(1, \pm\left(\frac{M^2}{\alpha^2} - 1\right)^{\frac{1}{2}}\right), \tag{22}$$

where
$$\alpha = \sin\omega_c t_1 \pm (M^2 - 1)^{\frac{1}{2}} \cos\omega_c t_1. \tag{23}$$

3.6. *A body accelerates in a straight path*

The equations may be applied to a body whose velocity varies with time, and in this final sub-section the simple case of a two-dimensional body moving with a constant acceleration in a horizontal plane is considered. A body with a Mach number M_2 begins to accelerate at time t_2 and we look at the wave system which has developed at time t, when the Mach number is M.

Equations (10), (14) and (15) give the following equation for the phase configuration:

$$\frac{\mathbf{r}}{a_0 T} = \frac{t_1}{T}\left[\frac{(M - M_2)\,t_1}{2T} + M_2, 0\right]$$
$$- \frac{M_1}{G(1 - M_1^2 C)}\frac{\phi}{NT}\{(1 - G^2), \pm G\,[G^2(M_1^2 - 1) + (1 - M_1^2 C)]^{\frac{1}{2}}\}, \tag{24}$$

where G, t_1 and M_1 are related by the equations

$$G^2\left(\frac{\phi}{NT}\right)M_1^2 - G\left(1 - \frac{t_1}{T}\right)(1 - M_1^2 C) - \frac{\phi}{NT} = 0 \tag{25}$$

and
$$M_1 = (M - M_2)\,t_1/T + M_2 \tag{26}$$

with
$$T = (t - t_2). \tag{27}$$

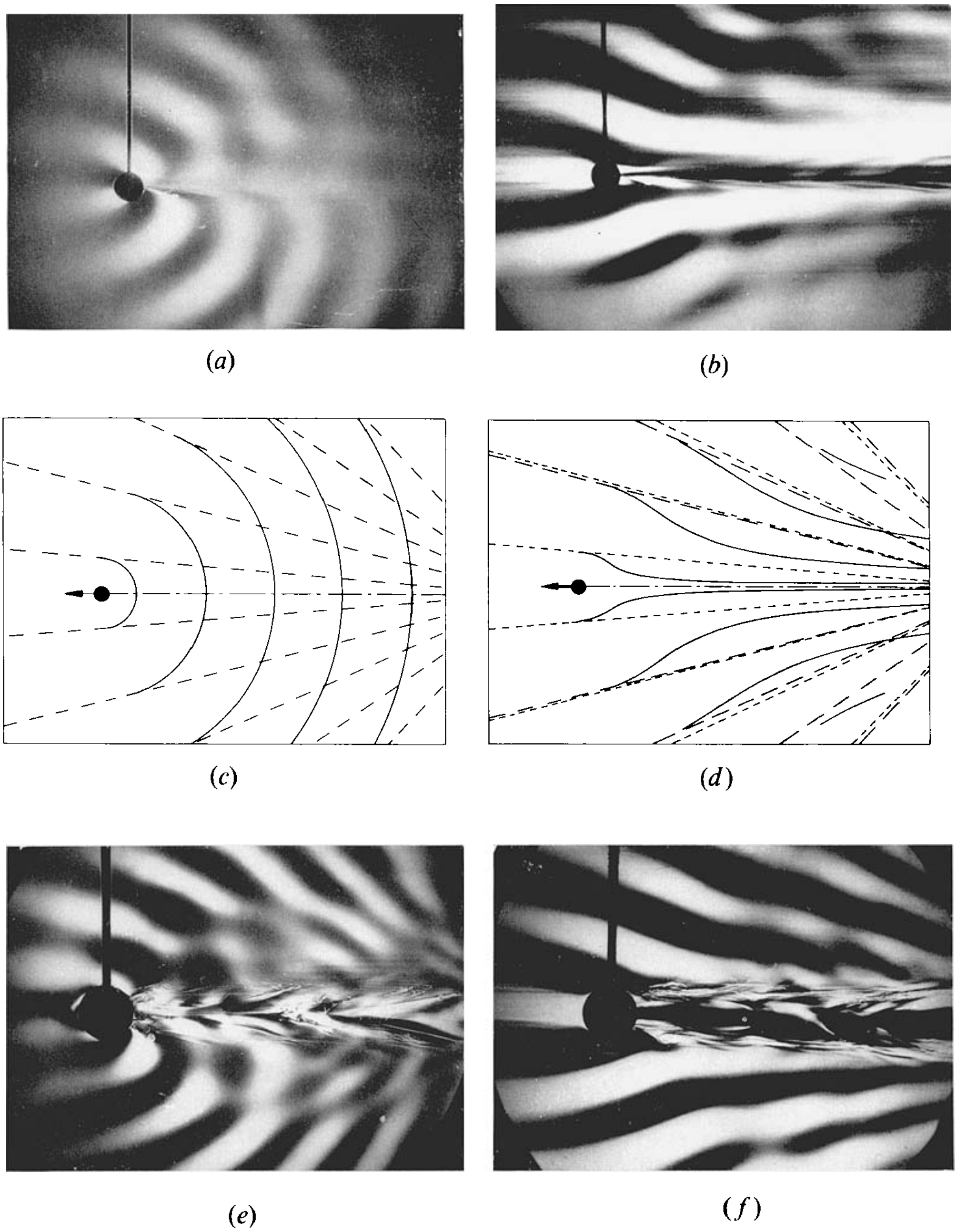


FIGURE 10. A sphere moves horizontally with a constant velocity of 13 mm/s. The schlieren photographs show a sphere of 25 mm diameter in (*a*) and (*b*), and of 50 mm in (*e*) and (*f*). A vertical knife edge was used for (*a*) and (*e*) and a horizontal knife edge for (*b*) and (*f*). The theoretical solution in the plane $y = 0$ is shown in (*c*) and the solution corresponding to $y/\phi = 2/\pi$ in (*d*). The solid lines are steady waves and the broken lines are impulsive waves.

The waves evaluated from these equations would be superimposed on the pseudo-steady wave system which was generated before t_2 [calculated from (18) and (19)] together with the impulsive wave system generated at time t_2.

In figure 17, lines of constant phase calculated from (24)–(27) are shown for a body which started from rest and has reached $M = 2$. The phase ϕ is given by

$$\phi = 2\pi NTK^2 \times 10^{-3}$$

and the waves are plotted for particular values of K. Thus, as expected, the lower the acceleration the larger the number of waves present. The higher the Mach number the larger is the wave spacing. The wave shown as the solid line in figure 17 is the pure acoustic wave which would result from motion in a homogeneous fluid.

4. Conclusions

Equations have been derived for the internal waves and gravity-modified acoustic waves which develop around a body moving on an arbitrary path in a compressible fluid with a constant buoyancy frequency. The wave patterns have been calculated using the radiation condition during the computation and the wavenumber surfaces have not been used explicitly. Solutions have been obtained for several cases including the two limiting cases of a sphere moving in an incompressible fluid and a two-dimensional body moving supersonically in a compressible fluid which is not stratified. The former has been shown to agree with experiments in stratified brine and the latter with the previous calculations of Lilley *et al.* (1953).

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REFERENCES

Cole, J. D. & Greiffinger, C. 1969 *J. Geophys. Res.* **74**, 3693.
Hines, C. O. 1960 *Can. J. Phys.* **38**, 1441.
Lighthill, M. J. 1965 *J. Inst. Math. Appl.* **1**, 1.
Lighthill, M. J. 1967 *I.A.U. Symp.* no. 28.
Lilley, G. M., Westley, R., Yates, A. H. & Busing, J. R. 1953 *J. Roy. Aero. Soc.* **57**, 400.
Liu, C. H. & Yeh, K. C. 1972 *Agard* CP 115, **8**, 1.
Moore, D. W. & Spiegel, E. A. 1964 *Astrophys. J.* **139**, 48.
Rarity, B. S. H. 1973 *Quart. J. Roy. Met. Soc.* **99**, 337.
Silcock, G. 1973 M.Sc. thesis, University of Manchester.
Stevenson, T. N. 1973 *J. Fluid Mech.* **60**, 759.
Stevenson, T. N. & Thomas, N. H. 1969 *J. Fluid Mech.* **36**, 505.

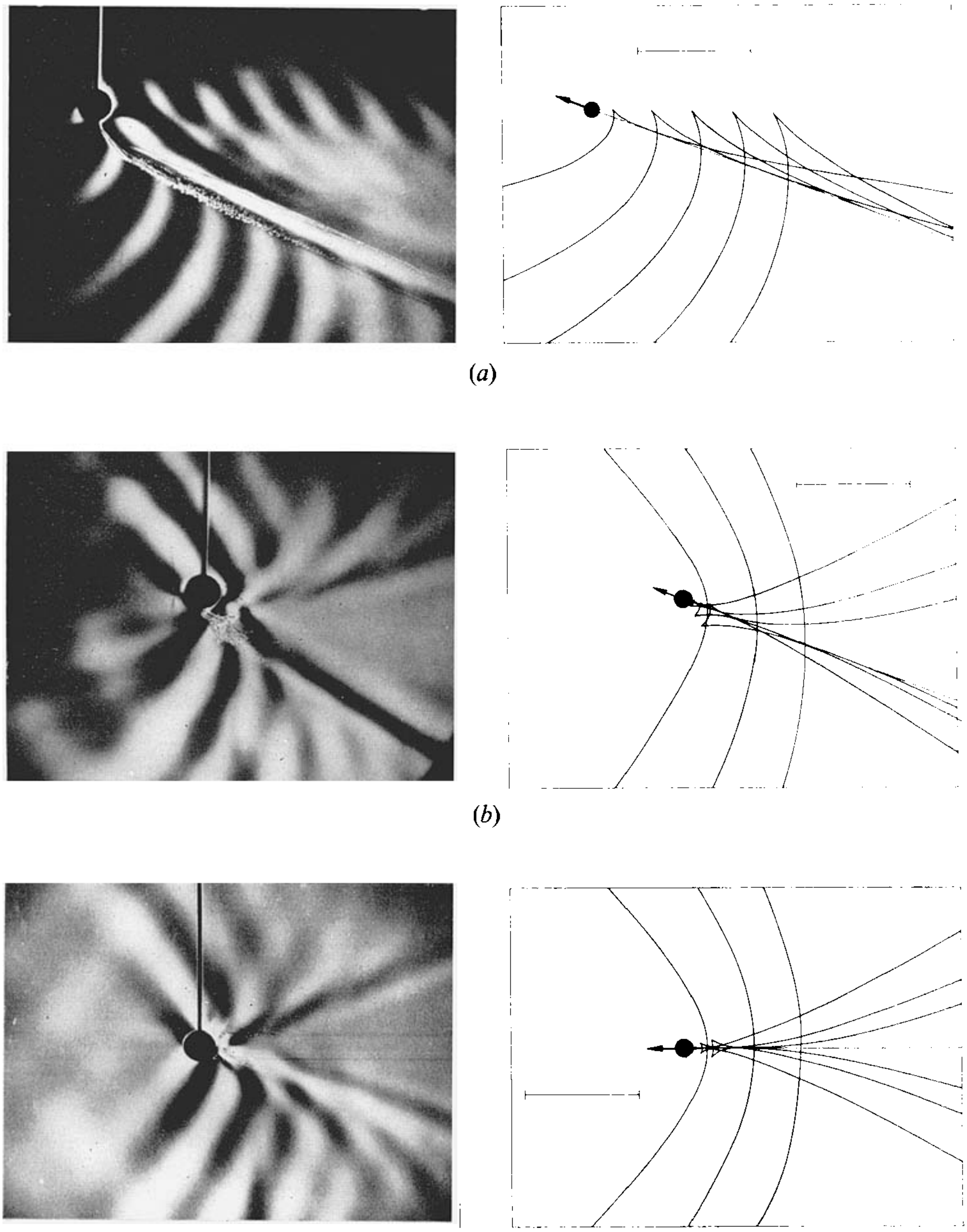

FIGURE 11. A sphere moves at constant velocity in the plane $y = 0$ at an angle θ to the horizontal. The theoretical solutions are in the plane $y = 0$. (*a*) $\theta = 20°$, $Q = 9{\cdot}2$ mm/s and $B = 0$. (*b*) $\theta = 20°$, $Q = 2{\cdot}9$ mm/s and $B = 0{\cdot}64$. (*c*) $\theta = 0°$, $Q = 2{\cdot}9$ mm/s and $B = 0{\cdot}61$. The scale marks are of length 100 mm.